AP Calculus AB

Volumes by Rotation

1) DISC $V = \pi \int R^2$

$V = \pi \int_0^2 (x^3)^2 dx$

$= 57.446$

2) WASHER

$V = \pi \int_0^2 R^2 - \pi \int_0^2 r^2$

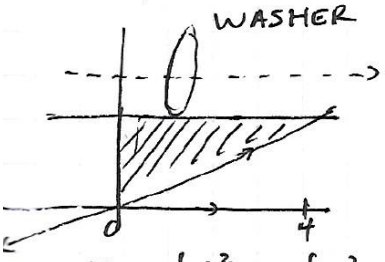
$= \pi \int_0^2 (2x-(-1))^2 dx - \pi \int_0^2 (1)^2 dx$

$= \pi \int_0^2 (2x+1)^2 dx - \pi \int_0^2 1\, dx$

$= 58.643$

3) $y = \frac{x}{2}$, $x=0$, $y=2$

WASHER



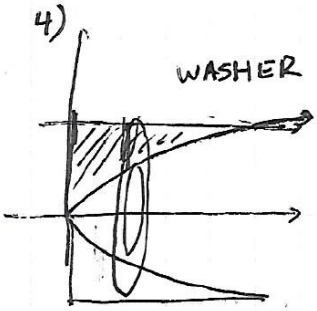
$V = \pi \int R^2 - \pi \int r^2$

$V = \pi \int_0^4 (3-\frac{x}{2})^2 dx - \pi \int_0^4 (3-2)^2 dx$

$= \pi \int_0^4 \left[(3-\frac{x}{2})^2 - 1\right] dx$

$= 41.8879$

4)

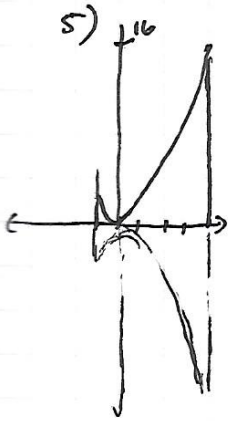


$V = \pi \int_0^4 R^2 - \pi \int_0^4 r^2$

$V = \pi \int_0^4 2^2 dx - \pi \int_0^4 (\sqrt{x})^2 dx$

$V = 8\pi$

5)



DISC

$V = \pi \int_{-1}^{4} (x^2)^2 dx$

$\pi \int_{-1}^{4} x^4 dx$

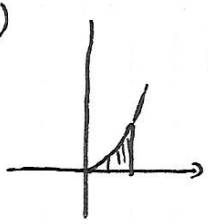
$\pi \left[\frac{1}{5}x^5 + C\right]_{-1}^{4}$

$\pi \left[\frac{1}{5}(4)^5 - \frac{1}{5}(-1)^5\right]$

$\pi \left[\frac{1024}{5} + \frac{1}{5}\right]$

$\boxed{205\pi}$

6)



DISC

$V = \pi \int_0^1 (x^4)^2 dx$

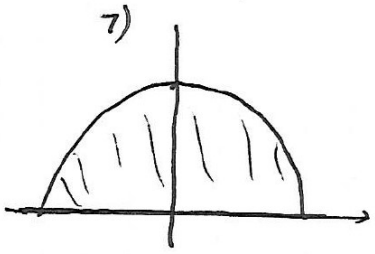
$\pi \int_0^1 x^8 dx$

$\pi \left[\frac{1}{9}x^9 + C\right]_0^1$

$\pi \left[\frac{1}{9} - 0\right]$

$\boxed{\frac{\pi}{9}}$

7)



DISC

$$V = \pi \int_{-3}^{3} \left(\sqrt{9-x^2}\right)^2 dx$$

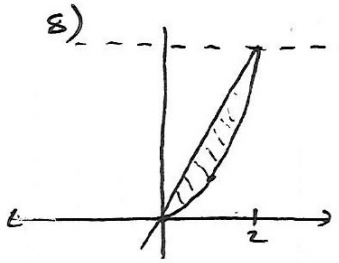
$$\pi \int_{-3}^{3} (9 - x^2)\, dx$$

$$= \pi \left[9x - \frac{1}{3}x^3 + C\right]_{-3}^{3}$$

$$= \pi \left[(27-9) - (-27+9)\right]$$

$$= \boxed{36\pi}$$

8)



WASHER

$$V = \pi \int_0^2 R^2 dx - \pi \int_0^2 r^2 dx$$

$$= \pi \int_0^2 \left[4 - x^2\right]^2 dx - \pi \int_0^2 \left[4 - 2x\right]^2 dx$$

$$= \pi \int_0^2 \left[(4-x^2)^2 - (4-2x)^2\right] dx$$

$$= \boxed{20.106}$$

9) $Y_1 = x^3 - 4x + 4 \quad Y_2 = 3x - 2$

$Y_1 = Y_2$ @ $x = -3,\ x = 1,\ x = 2$

$$\text{Area} = \int_{-3}^{1} (Y_1 - Y_2)\, dx + \int_{1}^{2} (Y_2 - Y_1)\, dx$$

$$= \boxed{32.75}$$